Poll: How big is infinity?

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Mark what's true.
(A) There are more real numbers than natural numbers.
(B) There are more rational numbers than natural numbers.
(C) There are more integers than natural numbers.
(D) pairs of natural numbers $>>$ natural numbers.

Same Size. Poll.

Two sets are the same size?

Same Size. Poll.

Two sets are the same size?

(A) Bijection between the sets.
(B) Count the objects and get the same number. same size.
(C) Counting to infinity is hard.

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(A), (B).
(C)?

Countable.

How to count?

Countable.

How to count?

0,

Countable.

How to count?

0, 1,

Countable.

How to count?

0, 1, 2,

Countable.

How to count?

0, 1, 2, 3,

Countable.

How to count?

0, 1, 2, 3, . . .

Countable.

How to count?

0, 1, 2, 3, $\ldots$

The Counting numbers.

Countable.

How to count?

0, 1, 2, 3, ...

The Counting numbers.
The natural numbers! N

Countable.

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0, 1, 2, 3, . . .

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Definition: S is **countable** if there is a bijection between S and some subset of N.

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If the subset of N is finite, S has finite **cardinality**.

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Definition: S is **countable** if there is a bijection between S and some subset of N.

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If the subset of N is infinite, S is **countably infinite**.

Countably infinite subsets.

Enumerating a set implies countable.
Corollary: Any subset T of a countable set S is countable.

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All countably infinite sets have the same cardinality.

Enumeration example.

All binary strings.

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$B = \{0, 1\}^*$.

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$B = \{\phi, 0, 1, 00, 01, 10, 11,$

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$B = \{\phi; , 0, 00, 000, 0000, ...\}$
Never get to 1.

More fractions?

Enumerate the rational numbers in order...

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Where is $1/2$ in list?

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Can't list in "order".

Pairs of natural numbers.

Consider pairs of natural numbers: $N \times N$

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For finite sets S_1 and S_2,
then $S_1 \times S_2$

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Enumerate in list:

Pairs of natural numbers.

Enumerate in list:
$(0,0)$,

Pairs of natural numbers.

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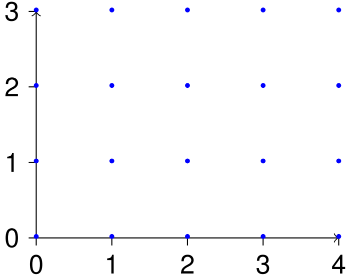

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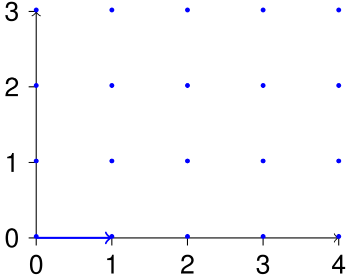

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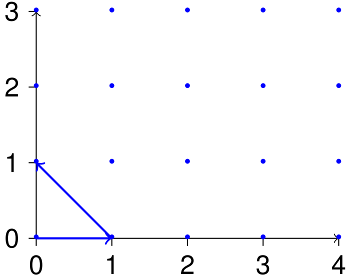

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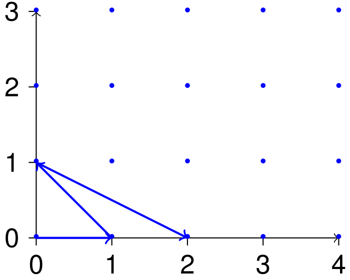

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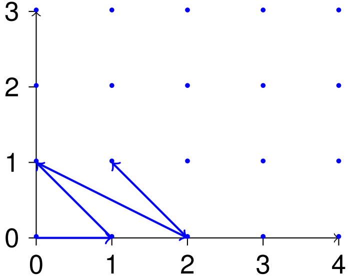

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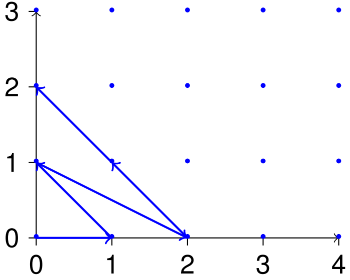

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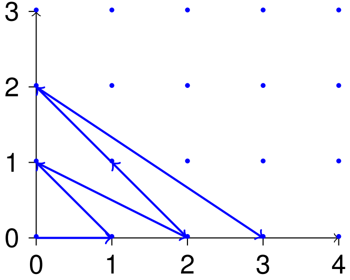

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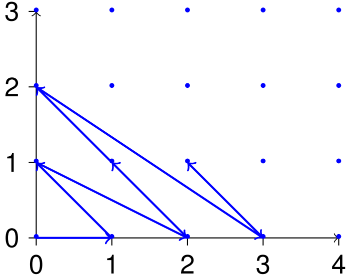

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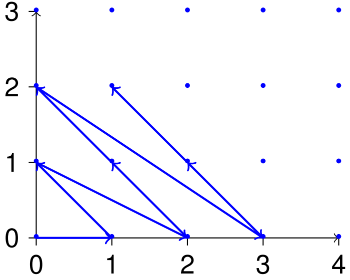

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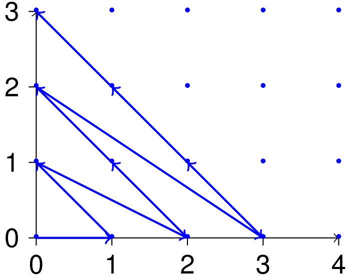

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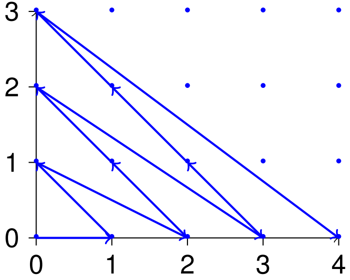

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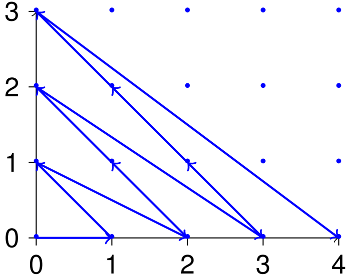

The pair (a,b), is in first $\approx (a+b+1)(a+b)/2$ elements of list!

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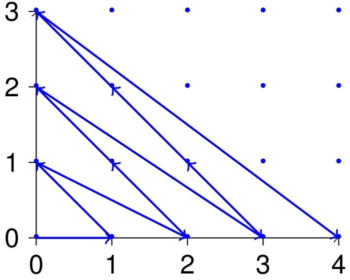

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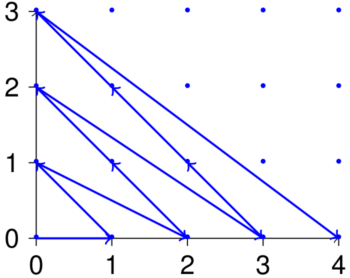

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Countably infinite.

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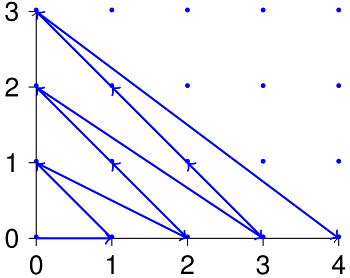

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Countably infinite.

Same size as the natural numbers!!

Poll.

Enumeration to get bijection with naturals?

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(A) Integers: First all negatives, then positives.
(B) Integers: By absolute value, break ties however.
(C) Pairs of naturals: by sum of values, break ties however.
(D) Pairs of naturals: by value of first element.
(E) Pairs of integers: by sum of values, break ties.
(F) Pairs of integers: by sum of absolute values, break ties.

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Enumeration to get bijection with naturals?

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(B),(C), (F).

Rationals?

Positive rational number.

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Lowest terms: a/b

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Negative rationals are countable.

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First negative, then nonegative

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First negative, then nonegative ??? No!

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    Interleave Streams in 61A

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 Interleave Streams in 61A

The rationals are countably infinite.

Real numbers..

Real numbers are same size as integers?

The reals.

Are the set of reals countable?

The reals.

Are the set of reals countable?

Lets consider the reals $[0, 1]$.

The reals.

Are the set of reals countable?

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Each real has a decimal representation.

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Lets consider the reals $[0,1]$.

Each real has a decimal representation.
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Subset $[0, 1]$ is not countable!!

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If reals are countable then so is $[0, 1]$.

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6. Contradiction.

Another diagonalization.

The set of all subsets of N.

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Example subsets of N: $\{0\}$,

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If ith set in L does not contain i, $i \in D$.

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Theorem: The set of all subsets of N is not countable.

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L does not contain all subsets of *N*.

Contradiction.

Theorem: The set of all subsets of *N* is not countable.
(The set of all subsets of *S*, is the **powerset** of *N*.)

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Mark parts of proof.

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(A) Integers are larger than naturals cuz obviously.
(B) Integers are countable cuz, interleaving bijection.
(C) Reals are uncountable cuz obviously!
(D) Reals can't be in a list: diagonal number not on list.
(E) Powerset in list: diagonal set not in list.

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(B), (C)?, (D), (E)

The Continuum hypothesis.

There is no set with cardinality between the naturals and the reals.

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First of Hilbert's problems!

Cardinalities of uncountable sets?

Cardinality of $[0, 1]$ smaller than all the reals?

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$$f(x) = \begin{cases} x + \frac{1}{2} & 0 \le x \le 1/2 \\ \frac{1}{4x} & x > 1/2 \end{cases}$$

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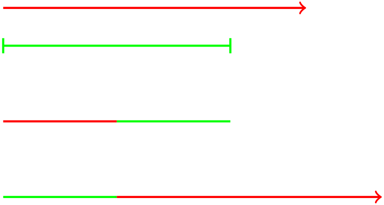

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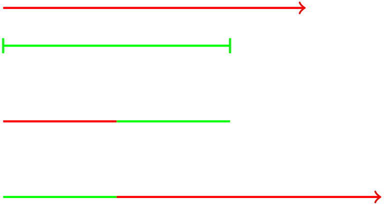

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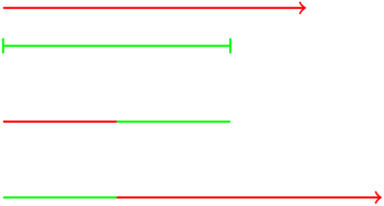

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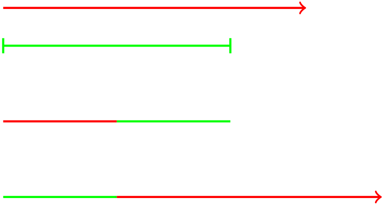

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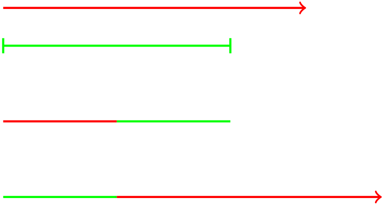

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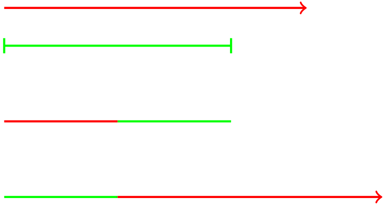

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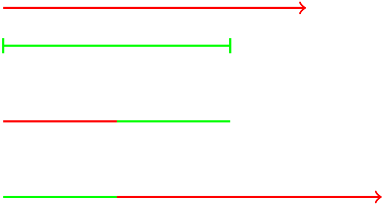

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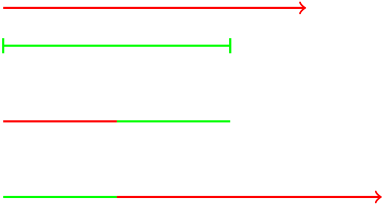

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$[0,1]$ is same cardinality as nonnegative reals!

Generalized Continuum hypothesis.

There is no infinite set whose cardinality is between the cardinality of an infinite set and its power set.

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The powerset of a set is the set of all subsets.

Resolution of hypothesis?

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Gödel. 1940.
Can't use math!

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(A) Barber not Mark. Barber shaves Mark.
(B) Mark shaves the Barber.
(C) Barber doesn't shave themself.
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Its all true.

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Its all true. It's all a problem.

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Recall: powerset of the naturals is not countable.

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Goedel:
Any set of axioms is either

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See Logicomix by Doxiaidis, Papadimitriou (was professor here), Papadatos, Di Donna.

Is it actually useful?

Write me a program checker!

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Check that the compiler works!

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How about.. Check that the compiler terminates on a certain input.

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$HALT(P, I)$

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Program is a text string.
Text string can be an input to a program.

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Run P on I and check!

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How long do you wait?

Implementing HALT.

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Determines if $P(I)$ (P run on I) halts or loops forever.

Run P on I and check!

How long do you wait?

Something about infinity here, maybe?

Halt does not exist.

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$HALT(P, I)$

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Theorem: There is no program HALT.

Halt does not exist.

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Proof: Yes!

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Yes! No!...

What is he talking about?

Yes! No!...

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(A) He is confused.
(B) Diagonalization.
(C) Welch-Berlekamp
(D) Professor is just strange.

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(B)

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(B) and (D)

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(B) and (D) maybe? and maybe (A).

Professor does me some love Welch-Berlekamp though!

Halt and Turing.

Proof:

Halt and Turing.

Proof: Assume there is a program $HALT(\cdot, \cdot)$.

Halt and Turing.

Proof: Assume there is a program *HALT*$(\cdot, \cdot)$.

Turing(P)

Halt and Turing.

Proof: Assume there is a program $HALT(\cdot, \cdot)$.

Turing(P)
1. If HALT(P,P) ="halts", then go into an infinite loop.

Halt and Turing.

Proof: Assume there is a program $HALT(\cdot,\cdot)$.

Turing(P)
1. If HALT(P,P) =“halts”, then go into an infinite loop.
2. Otherwise, halt immediately.

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There is text that "is" the program HALT.

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Can run Turing on Turing!

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Does Turing(Turing) halt?

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Turing(Turing) halts
$\Longrightarrow$ then HALTS(Turing, Turing) $=$ halts

Halt and Turing.

Proof: Assume there is a program $HALT(\cdot, \cdot)$.

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Can run Turing on Turing!

Does Turing(Turing) halt?

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Another view of proof: diagonalization.

Any program is a fixed length string.

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P_2	L	L	H	$\cdots$
P_3	L	H	H	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

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Programs?

What are programs?

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(A) Instructions.
(B) Text.
(C) Binary String.
(D) They run on computers.

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All are correct.

Proof play by play.

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We are so smart!

Wow, that was easy!

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We should be famous!

No computers for Turing!

In Turing's time.

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Adding machines.

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Concept of program as data wasn't really there.

Turing machine.

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A Turing machine.
– an (infinite) tape with characters

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Turing: AI, self modifying code, learning...

Turing and computing.

Just a mathematician?

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Simulated the program by hand to play chess.

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The polish machine...the *bomba.*

Computing on top of computing...

Computer, assembly code, programming language, browser, html, javascript..

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Computer, assembly code, programming language, browser, html, javascript..

We can't get enough of building more Turing machines.

Undecidable problems.

Does a program, P, print “Hello World”?

Undecidable problems.

Does a program, P, print "Hello World"?
How?

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Does a program, P, print "Hello World"?
How? What is P?

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Find exit points and add statement: **Print** “Hello World.”

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The answer is yes or no.

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Is there an integer solution to $x^n + y^n = 1$?
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The answer is yes or no. This “problem” is not undecidable.

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Does a program, P, print “Hello World”?
How? What is P? Text!!!!!!

Find exit points and add statement: **Print** “Hello World.”

Can a set of notched tiles tile the infinite plane?
Proof: simulate a computer. Halts if finite.

Does a set of integer equations have a solution?
Example: “ $x^n + y^n = 1$?”
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Is there an integer solution to $x^n + y^n = 1$?
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Turing: personal.

Tragic ending...

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- British Government apologized (2009) and pardoned (2013).

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Program is text, so we can pass it to itself,
or refer to self.

Summary: decidability.

Computer Programs are an interesting thing.

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Computation is a lens for other action in the world.

Kolmogorov Complexity, Google, and CS70

Of strings, s.

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Of strings, *s*.

Minimum sized program that prints string *s*.

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What Kolmogorov complexity of a string of 1,000,000, one's?

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for $i = 1$ to n: print '1'.

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What is the minimum I need to know (remember) to know stuff.

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Radius of the earth?

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Conceptualization.
Reason and understand an argument and you can generate a lot.

Calculus

What is the first half of calculus about?

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The slope of a tangent line to a function at a point.

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Slope is rise/run. Oh, yes: $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

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Chain rule?

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But...but...

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The slope of a tangent line to a function at a point.

Slope is rise/run. Oh, yes: $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

Chain rule? Derivative of a function composition.

Intuition: composition of two linear functions?

$f(x) = ax$, $g(x) = bx$. $f(g(x)) = ab\,x$. Slope is ab.

Multiply slopes!

$(f(g(x))' = f'(\cdot)g'(\cdot)$

But...but...

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Product Rule.

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Any concept:

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A quick argument from basic concept of slope of a tangent line.

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Perhaps.

Derivative of sine?

$\sin(x)$.

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What is x? An angle in radians.

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Let's call it θ and do derivative of $\sin\theta$.

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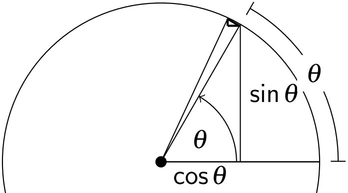

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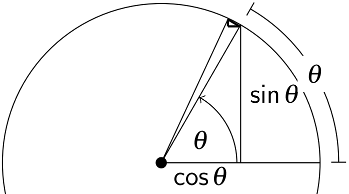

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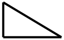

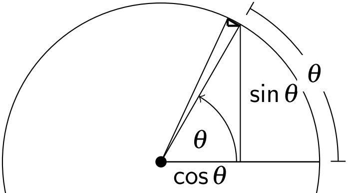

Rise. Similar triangle!!!

Fundamental Theorem of Calculus.

Conceptual: Height times Width = Area.

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Useful?

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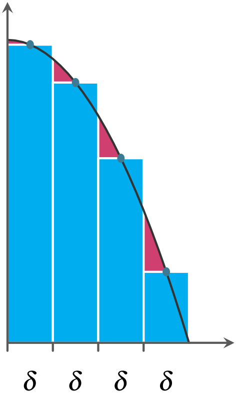
Derivative (rate of change) of Area (Integral) under curve, is height of curve.

Calculus



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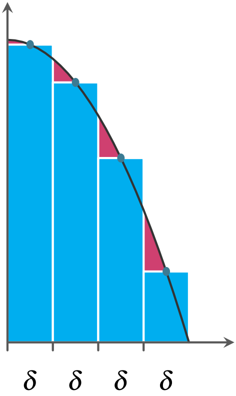
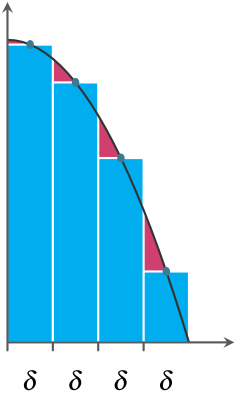


Riemann Sum/Integral: $\int_a^b f(x)dx = lim_{\delta \to 0} \sum_i \delta f(a_i)$

Calculus



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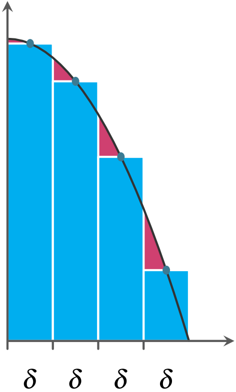

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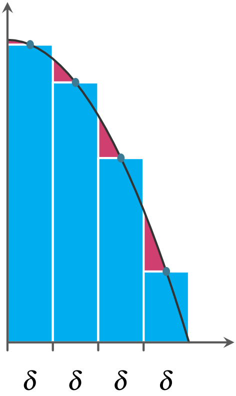

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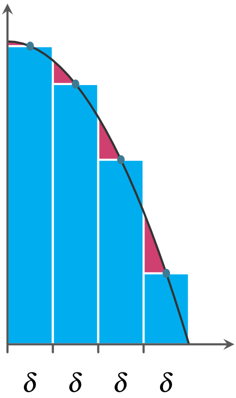

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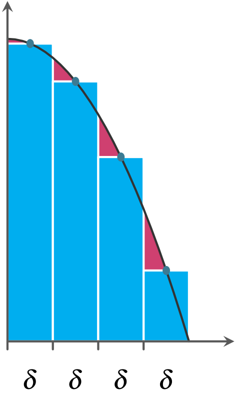

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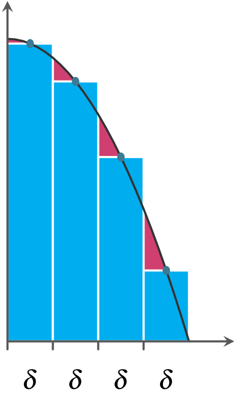

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"Area ($F(\cdot)$) under $f(x)$ grows at x, $F'(x)$, by $f(x)$"
Thus $F'(x) = f(x)$.

Arguments, reasoning.

What you know: slope, limit.

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 Plus: definition.

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yields calculus.

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Knowing how to program

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Knowing how to program plus some syntax (google) gives the ability to program.

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Knowing how to reason plus some definition

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Discrete Math: basics are counting, how many, when are two sets the same size?

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Probability: division.

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Discrete Math: basics are counting, how many, when are two sets the same size?

Probability: division.

...plus reasoning.

CS 70 : ideas.

Induction

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Induction $\equiv$ every integer has a next one.

CS 70 : ideas.

Induction $\equiv$ every integer has a next one. Graph theory.

Number of edges is sum of degrees.

$\Delta + 1$ coloring. Neighbors only take up Δ.

Connectivity plus connected components.

Eulerian paths: if you enter you can leave.

Euler's formula: tree has $v-1$ edges and 1 face plus each extra edge makes additional face.

$v - 1 + (f - 1) = e$

CS 70 : ideas.

Number theory.

A divisor of x and y divides $x - y$.

The remainder is always smaller than the divisor.

$\Longrightarrow$ Euclid's GCD algorithm.

Multiplicative Inverse.

Fermat's theorem from function with inverse is a bijection.

Gives RSA.

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$\implies$ Euclid's GCD algorithm.
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Error Correction.
(Any) Two points determine a line.
(well, and d points determine a degree $d+1$-polynomials.
Cuz, factoring.
Find line by linear equations.
If a couple are wrong, then multiply them by zero, i.e., Error polynomial.

CS70 and your future?

What's going on?

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Define. Understand properties. And build from there.

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....and you will pursue probability in this course.