70: Discrete Math and Probability Theory

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Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
Logic and Proofs!
Induction $\equiv$ Recursion.

What can computers do?
Work with discrete objects.
Discrete Math $\implies$ immense application.

Computers learn and interact with the world?
E.g. machine learning, data analysis, robotics, ...
Probability!

My hopes and dreams.

You learn to think more clearly and more powerfully.

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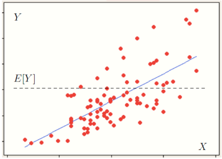
You learn to think more clearly and more powerfully.

..And to deal clearly with uncertainty itself.

Probability Unit

- How can we predict unknown future events (e.g., gambling profit, next week rainfall, traffic congestion, …)?
  - Constructive Models: Model the overall system (including the sources of uncertainty).
    - For modeling uncertainty, we'll develop probabilistic models and techniques for analyzing them.
  - Deductive Models: Extract the "trend" from the previous outcomes (e.g., linear regression).



Learning.

Veritassium on Khan

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Confusion is the sweat of learning.

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Confusion is the sweat of discovery.

Metacogition.

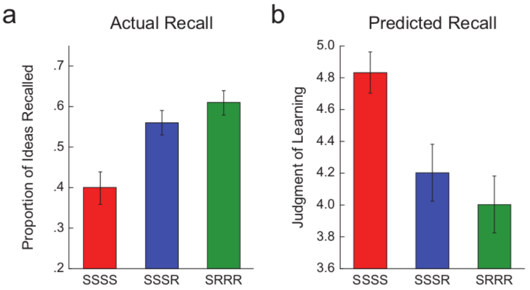

Fig. 1. Final recall (a) after repeatedly studying a text in four study periods (SSSS condition), reading a text in three study periods and then recalling it in one retrieval period (SSSR condition), or reading a text in one study period and then repeatedly recalling it in three retrieval periods (SRRR condition). Judgments of learning (b) were made on a 7-point scale, where 7 indicated that students believed they would remember material very well. The data presented in these graphs are adapted from Experiment 2 of Roediger and Karpicke (2006b). The pattern of students' metacognitive judgments of learning (predicted recall) was exactly the opposite of the pattern of students' actual long-term retention.

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Notes cover material. Discussion. Vitamins. Homework. Study.

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My advice to TA's.

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What should you do?

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What should you do?
Where does your understanding get iffy?

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What should you do?
  Where does your understanding get iffy?
    Explain what you understand, then say what you don't.

Advice from (former) TA's

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Head TA Richard:
"carefully review the homework solutions after they are released and understand them to the point of being able to replicate them without needing to reference them."

Admin

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Announcements, logistics, critical advice.

Wason's experiment:1

Suppose we have four cards on a table:

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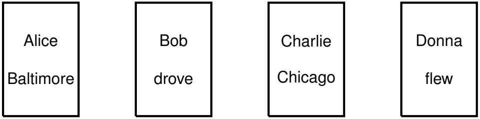
- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.



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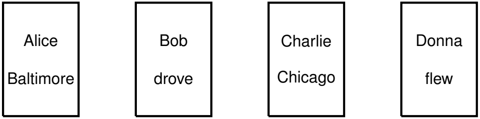

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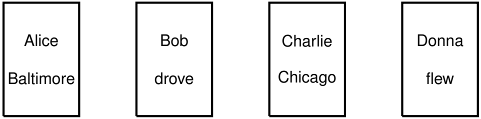

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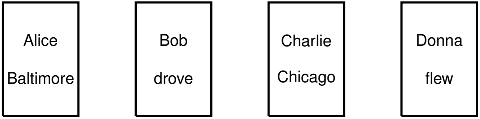

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Today: Note 1.

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The language of proofs!

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The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational
2+2 = 4
2+2 = 3
826th digit of pi is 4
Johnny Depp is a good actor
Any even >2 is sum of 2 primes
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I love you.		

Propositions: Statements that are true or false.

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2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4	**Proposition**	False
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Put propositions together to make another...

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Propositions:

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Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q)$

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True if both P and Q are True .

"$P \vee Q$" is True if $\geq$ one of P or Q is True .

P	Q	$P \wedge Q$
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T	F	F
F	T	F
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T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q \qquad\qquad \neg(P \vee Q) \quad \equiv \quad \neg P \wedge \neg Q$

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Is $(T \wedge Q) \equiv Q$?

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes?

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes!

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

What is $(F \vee Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

What is $(F \vee Q)$? Q

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Distributive?

$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$?

Simplify: $(T \land Q) \equiv Q$,

Distributive?

$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$?

Simplify: $(T \land Q) \equiv Q$, $(F \land Q) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.
P is False .
LHS: $F \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.
P is False .
LHS: $F \wedge (Q \vee R) \equiv F$.
RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$,

Distributive?

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P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
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P is False .
LHS: $F \wedge (Q \vee R) \equiv F$.
RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.
P is False .
LHS: $F \wedge (Q \vee R) \equiv F$.
RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:
$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Foil 2:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Foil 2:

$(A \wedge B) \vee (C \wedge D) \equiv (A \vee C) \wedge (A \vee D) \wedge (B \vee C) \wedge (B \vee D)$?

Implication.

$P \implies Q$ interpreted as

Implication.

$P \implies Q$ interpreted as

If P, then Q.

Implication.

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True Statements: P, $P \implies Q$.

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Examples:

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Examples:

Statement: If you stand in the rain, then you'll get wet.

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
Q = "you will get wet"

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$P \implies Q$ interpreted as

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Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
Q = "you will get wet"
Statement: "Stand in the rain"

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
Q = "you will get wet"
Statement: "Stand in the rain"
Can conclude: "you'll get wet."

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

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Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
Q = "you will get wet"
Statement: "Stand in the rain"
Can conclude: "you'll get wet."

Statement:
If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
Q = "you will get wet"
Statement: "Stand in the rain"
Can conclude: "you'll get wet."

Statement:
If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

P = "a right triangle has sidelengths $a \leq b \leq c$",

Implication.

$P \implies Q$ interpreted as

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Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
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Statement: "Stand in the rain"
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If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

P = "a right triangle has sidelengths $a \leq b \leq c$",
Q = "$a^2 + b^2 = c^2$".

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing

Non-Consequences/consequences of Implication

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P False means

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

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False implies nothing
P False means Q can be True

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.

Non-Consequences/consequences of Implication

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If chemical plant pollutes river, fish die.

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If fish die, did chemical plant pollute river?

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If fish die, did chemical plant pollute river?

Not necessarily.

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If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and Q are True does not mean P is True

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If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and Q are True does not mean P is True

Be careful!

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

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False implies nothing
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Anything implies true.
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If fish die, did chemical plant pollute river?

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Be careful!

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Non-Consequences/consequences of Implication

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Implication and English.

$P \implies Q$

Poll.

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  Example: It is necessary that $n > 3$ for $n > 4$.

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	
F	T	
F	F	

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These two propositional forms are logically equivalent!

Contrapositive, Converse

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- **Definition:** If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.
  (Logically Equivalent: $\iff$.)

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

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- $C(x) \implies F(x)$.

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- $R(x) =$ "$x > 2$"
- $G(n) =$ "n is even and the sum of two primes"
- Remember Wason's experiment!
  $F(x) =$ "Person x flew."
  $C(x) =$ "Person x went to Chicago
- $C(x) \implies F(x)$. Theory from Wason's.
  If person x goes to Chicago then person x flew.

Next:

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) =$ "x is even"
Same as boolean valued functions from 61A!

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Next: Statements about boolean valued functions!!

Quantifiers..

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Wait!

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Wait! What is $\mathbb{N}$?

Quantifiers: universes.

Proposition: "For all natural numbers n**,** $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$**."**

Proposition has **universe**:

Quantifiers: universes.

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Proposition has **universe**: "the natural numbers".

Universe examples include..

- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).
- $\mathbb{Z} = \{\ldots, -1, 0, \ldots\}$ (integers)
- $\mathbb{Z}^+$ (positive integers)
- $\mathbb{R}$ (real numbers)
- Any set: $S = \{Alice, Bob, Charlie, Donna\}$.
- See note 0 for more!

Other proposition notation(for discussion):
"$d|n$" means d divides n

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$2|4$?

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Back to: Wason's experiment:1

Theory:

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$Chicago(A)$ = False .

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$\mathit{Chicago}(A)$ = False . Do we care about $\mathit{Flew}(A)$?

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So $Chicago(Bob)$ must be False .

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Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

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$Flew(D)$ = True .

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$Flew(D)$ = True . Do we care about $Chicago(D)$?

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since $Chicago(A)$ is False ,

$Flew(B)$ = False . Do we care about $Chicago(B)$?
Yes. $Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B)$.
So $Chicago(Bob)$ must be False .

$Chicago(C)$ = True . Do we care about $Flew(C)$?
Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

$Flew(D)$ = True . Do we care about $Chicago(D)$?
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Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she/they flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

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Only have to turn over cards for Bob and Charlie.

More for all quantifiers examples.

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- “doubling a number always makes it larger”

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Later we may omit universe if clear from context.

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Theorem: $(\forall n \in N) \neg(\exists a, b, c \in N) (n \geq 3 \implies a^n + b^n = c^n)$

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Next Time: proofs!