

1 Continuous Intro

(a) Is

$$f(x) = \begin{cases} 2x, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

a valid density function? Why or why not? Is it a valid CDF? Why or why not?

(b) Calculate the PDF $f_X(x)$, along with $\mathbb{E}[X]$ and $\text{Var}(X)$ if the CDF of X is

$$F_X(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x}{\ell}, & 0 \leq x \leq \ell, \\ 1, & x \geq \ell \end{cases}$$

(c) Suppose X and Y are independent and have densities

$$f_X(x) = \begin{cases} 2x, & 0 \leq x \leq 1, \\ 0, & \text{otherwise,} \end{cases} \quad f_Y(y) = \begin{cases} 1, & 0 \leq y \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

What is their joint distribution? (Hint: for parts (c) and (d), we can use independence in much the same way that we did in discrete probability)

(d) Calculate $\mathbb{E}[XY]$ for the X and Y in part (c).

2 Uniform Distribution

You have two fidget spinners, each having a circumference of 10. You mark one point on each spinner as a needle and place each of them at the center of a circle with values in the range $[0, 10)$ marked on the circumference. If you spin both (independently) and let X be the position of the first spinner's mark and Y be the position of the second spinner's mark, what is the probability that $X \geq 5$, given that $Y \geq X$?

3 Lunch Meeting

Alice and Bob agree to try to meet for lunch between 12 PM and 1 PM at their favorite sushi restaurant. Being extremely busy, they are unable to specify their arrival times exactly, and can say only that each of them will arrive (independently) at a time that is uniformly distributed within the hour. In order to avoid wasting precious time, if the other person is not there when they arrive they agree to wait exactly fifteen minutes before leaving. What is the probability that they will actually meet for lunch? (hint: Sketch the joint distribution of the arrival times of Alice and Bob. What parts of the distribution corresponds to them meeting for lunch?)