1 Balls and Bins

Suppose you throw n balls into n labeled bins one at a time.

(a) What is the probability that the first bin is empty?

(b) What is the probability that the first k bins are empty?

(c) Let A be the event that at least k bins are empty. Notice that there are m subsets of k bins out of the total n bins. If we assume A_i is the event that the ith set of k bins is empty. Then we can write A as the union of A_i's:

$$A = \bigcup_{i=1}^{m} A_i.$$

Use the union bound to give an upper bound on the probability $\mathbb{P}[A]$.

(d) What is the probability that the second bin is empty given that the first one is empty?

(e) Are the events that "the first bin is empty" and "the first two bins are empty" independent?

(f) Are the events that "the first bin is empty" and "the second bin is empty" independent?

2 Head Count

Consider a coin with $\mathbb{P}[\text{Heads}] = 2/5$. Suppose you flip the coin 20 times, and define X to be the number of heads.

(a) What is $\mathbb{P}[X = k]$, for some $0 \le k \le 20$?

(b) Name the distribution of X and what its parameters are.

(c) What is $\mathbb{P}[X \ge 1]$? Hint: You should be able to do this without a summation.

(d) What is $\mathbb{P}[12 \le X \le 14]$?

3 Exploring the Geometric Distribution

Suppose $X \sim \text{Geometric}(p)$ and $Y \sim \text{Geometric}(q)$ are independent. Find the distribution of $\min(X, Y)$ and justify your answer.