

1 Venn Diagram

Out of 1,000 computer science students, 400 belong to a club (and may work part time), 500 work part time (and may belong to a club), and 50 belong to a club and work part time.

(a) Suppose we choose a student uniformly at random. Let C be the event that the student belongs to a club and P the event that the student works part time. Draw a picture of the sample space Ω and the events C and P .

(b) What is the probability that the student belongs to a club?

(c) What is the probability that the student works part time?

(d) What is the probability that the student belongs to a club AND works part time?

(e) What is the probability that the student belongs to a club OR works part time?

2 Flippin' Coins

Suppose we have an unbiased coin, with outcomes H and T , with probability of heads $\mathbb{P}[H] = 1/2$ and probability of tails also $\mathbb{P}[T] = 1/2$. Suppose we perform an experiment in which we toss the coin 3 times. An outcome of this experiment is (X_1, X_2, X_3) , where $X_i \in \{H, T\}$.

(a) What is the *sample space* for our experiment?

(b) Which of the following are examples of *events*? Select all that apply.

- $\{(H, H, T), (H, H), (T)\}$
- $\{(T, H, H), (H, T, H), (H, H, T), (H, H, H)\}$
- $\{(T, T, T)\}$
- $\{(T, T, T), (H, H, H)\}$
- $\{(T, H, T), (H, H, T)\}$

(c) What is the complement of the event $\{(H, H, H), (H, H, T), (H, T, H), (H, T, T), (T, T, T)\}$?

(d) Let A be the event that our outcome has 0 heads. Let B be the event that our outcome has exactly 2 heads. What is $A \cup B$?

(e) What is the probability of the outcome (H, H, T) ?

(f) What is the probability of the event that our outcome has exactly two heads?

(g) What is the probability of the event that our outcome has at least one head?

3 Sampling

Suppose you have balls numbered $1, \dots, n$, where n is a positive integer ≥ 2 , inside a coffee mug. You pick a ball uniformly at random, look at the number on the ball, replace the ball back into the coffee mug, and pick another ball uniformly at random.

- (a) What is the probability that the first ball is 1 and the second ball is 2?

- (b) What is the probability that the second ball's number is strictly less than the first ball's number?

- (c) What is the probability that the second ball's number is exactly one greater than the first ball's number?

- (d) Now, assume that after you looked at the first ball, you did *not* replace the ball in the coffee mug (instead, you threw the ball away), and then you drew a second ball as before. Now, what are the answers to the previous parts?