

## 1 Party Tricks

You are at a party celebrating your completion of the CS 70 midterm. Show off your modular arithmetic skills and impress your friends by quickly figuring out the last digit(s) of each of the following numbers:

- (a) Find the last digit of  $11^{3142}$ .
- (b) Find the last digit of  $9^{9999}$ .
- (c) Find the last digit of  $3^{641}$ .

## 2 Modular Potpourri

Prove or disprove the following statements:

- (a) There exists some  $x \in \mathbb{Z}$  such that  $x \equiv 3 \pmod{16}$  and  $x \equiv 4 \pmod{6}$ .
  
- (b)  $2x \equiv 4 \pmod{12} \iff x \equiv 2 \pmod{12}$ .
  
- (c)  $2x \equiv 4 \pmod{12} \iff x \equiv 2 \pmod{6}$ .

### 3 Modular Inverses

Recall the definition of inverses from lecture: let  $a, m \in \mathbb{Z}$  and  $m > 0$ ; if  $x \in \mathbb{Z}$  satisfies  $ax \equiv 1 \pmod{m}$ , then we say  $x$  is an **inverse of  $a$  modulo  $m$** .

Now, we will investigate the existence and uniqueness of inverses.

- (a) Is 3 an inverse of 5 modulo 10?
- (b) Is 3 an inverse of 5 modulo 14?
- (c) Is each  $3 + 14n$  where  $n \in \mathbb{Z}$  an inverse of 5 modulo 14?
- (d) Does 4 have inverse modulo 8?
- (e) Suppose  $x, x' \in \mathbb{Z}$  are both inverses of  $a$  modulo  $m$ . Is it possible that  $x \not\equiv x' \pmod{m}$ ?

### 4 Fibonacci GCD

The Fibonacci sequence is given by  $F_n = F_{n-1} + F_{n-2}$ , where  $F_0 = 0$  and  $F_1 = 1$ . Prove that, for all  $n \geq 1$ ,  $\gcd(F_n, F_{n-1}) = 1$ .